RSA

Projects

Solving the maths problem: international perspectives on mathematics education

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Contents

The **RSA** in partnership with

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Executive summary

The Vorderman Report (2011) has reinvigorated debate about the state of mathematics education in England. The statistics it cites are stark: nearly 50% of students fail to achieve GCSE Mathematics in secondary education and England ranks amongst the lowest in the world for rates of students studying mathematics in upper secondary education. This report outlines the core choices facing mathematics education and builds on the recommendations of the Vorderman Report by examining which mathematics approaches and reforms have worked overseas. The paper does not recommend directly importing practice from either of the case study sites, high-achieving Hong Kong or neighbouring Scotland, but rather that their experiences of wide-ranging reforms should inform debate in England.

Whilst the attainment of English students is at best stable, or even in decline, mathematical skills are growing in importance. Basic numeracy and quantitative skills are increasingly necessary in all jobs and life-skills, for tasks including budgeting and data-handling. And the changing nature of the international economy means that maths skills and knowledge are in higher demand than ever. Science, Technology, Engineering and Mathematics (STEM) industries are becoming increasingly central to economic competitiveness and growth and will provide many of the jobs of tomorrow for young people (Royal Society, 2011).

The reasons for England's relative underperformance in both attainment and progression are numerous. A growing culture of league-tables and 'teaching to the test' (or, performativity) in schools means that decisions about how mathematics is taught and assessed in schools are not always in the best interests of students and have actually resulted in counter-productive practices such as multiple and early entry to GCSE. With insufficient emphasis on functional and basic skills, and post-16 pathways that are usually limited to academic GCE Mathematics or GCSE re-takes, the current range of qualifications has further neglected learner needs and turned many students away from the subject.

50%

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There is a range of curriculum and qualification reforms already being explored and in some cases piloted in England. These include the Vorderman Report endorsed 'linked pair' GCSE qualification which makes clearer differentiation between functional and academic skills; the recommendation to make some form of mathematics compulsory until 18; and the widening of upper secondary mathematics pathways to include a Mature GCSE and a Maths for Citizenship qualification.

Our international case studies similarly emphasise a distinction between academic and functional mathematics: junior secondary education in Hong Kong incorporates both functional and academic content for all students. The favoured approach in Scotland is splitting mathematics into two separate qualifications (Mathematics and Lifeskills Mathematics), with one focusing on academic mathematics and the other on functional and workplace skills. The case studies also emphasise the importance of being as flexible as possible to meet learner needs, in sharp contrast to the regimented and examination-driven approach that characterises England's mathematics qualifications. For instance, some Scottish qualifications have no external examinations and the country's bi-level approach means students can move between qualifications easily depending on the extent of their progress.

In both Hong Kong and Scotland, upper secondary mathematics qualifications prioritise diversity in curriculum content over diversity in qualification-type. In fact, in both countries all students take the same qualifications (in Scotland the Higher and in Hong Kong the NSS). The Higher allows students to study the application of mathematics to real-life and the Hong Kong Diploma allows high-attaining students to specialise in calculus or algebra through the Extended Part. The key point being that in both cases – Hong Kong and Scotland – qualifications are available at different levels and incorporate relevance to the real-world in all qualifications, leaving mathematics in these countries more flexible and less niche.

Recommendations

More detail on the case studies and how they compare to mathematics education in England is available in the main body of the report but the approaches they outline which might inform or give strength to our own reforms and the recommendations of the Vorderman Report are as follows:

- Qualifications need to be flexible and bi-level to allow students to progress at their own speed and aim for the highest possible grade (as per National 4 and 5 in Scotland). On this basis the linked pair of mathematics GCSEs currently being piloted should be rolled out across England.
- Upper secondary education should offer more than re-takes for students who have not gained GCSE Mathematics
- Further consideration of making mathematics compulsory for all students in upper secondary education is needed and should draw on Hong Kong's NSS experience
- Mathematics qualifications in secondary and upper secondary education should incorporate functional *and* academic content (including arithmetic)
- Assessment arrangements should limit performativity, for instance through some element of teacher-led assessment for students studying foundation-level qualifications
- Too much diversity in qualification-type can be confusing for students, education-providers and employers. Diversity in curriculum content can be a better route for meeting diverse learner needs (as with the Scottish Higher and Hong Kong's NSS)
- Once the pilot and evaluation of the linked pair GCSEs is complete, the mathematics community should come together over a one or two day period to consider the big picture and learning from overseas, and develop definitive recommendations for the future of mathematics education in England.

The ideas and examples from home and overseas outlined in this paper should provide some food for thought, in some instances strengthening the case for intended reforms and in others providing new and sometimes challenging thinking. Both will be necessary to realise the goal of creating a 'world class mathematics education for all'.

Introduction

"Mathematical knowledge and skills are not in decline worldwide but are patterned differently across the globe. In Japan and China, more than 50% of degrees are awarded in STEM subjects compared to less than a quarter in the UK and only 16% in the United States"

(Kuenzi, 2008)

In the seven years since Adrian Smith's Inquiry into the failures of the mathematics system in England, little seems to have changed. In 2010 nearly 50% of students failed to obtain a C grade or above in GCSE Mathematics (Vorderman, 2011) and only 15% of students continued to study mathematics after 16 (Nuffield, 2010). The continuing difficulties that England experiences in delivering a strong mathematics education are re-visited in the Vorderman Report which outlines a series of reforms. Some far reaching, including the recommendation that mathematics becomes compulsory for all students until 18, whilst others including the 'linked pair' GCSE build on existing pilots.

Poor outcomes bring with them serious challenges. Most obvious is the effect on adult life skills: one in four adults are considered functionally innumerate meaning that up to 25% of the adult population lack numeracy skills essential for work and citizenship. But as the recent Vorderman Report (2011) has argued, England's poor maths rankings also affect broader economic and educational goals as Science, Technology, Engineering and Mathematics (STEM) industries become increasingly central to economic competitiveness and growth.

Mathematical knowledge and skills are not in decline worldwide but are patterned differently across the globe. In Japan and China, more than 50% of degrees are awarded in STEM subjects compared to less than a quarter in the UK and only 16% in the United States (Kuenzi, 2008). Maths students in Hong Kong are amongst the highest performing in the world, ranking fourth in the most recent PISA results and all students study mathematics until entering university or the workplace. Rates of maths study even vary within the United Kingdom. A quarter of Scottish students study mathematics until at least

18 compared with only 15% of English students (Nuffield, 2010). These international variations demonstrate that decline and disengagement from mathematics is not inevitable. Variations in curriculum, assessment and other education policies mean the study of mathematics is different depending on where you live.

This report explores different approaches to mathematics education in secondary and upper secondary education in the context of the Vorderman Report recommendations. The paper draws together recent research and policy work in the field to draw out key themes including why mathematics is important both for individual attainment and broader educational and economic goals and the main challenges that hinder England's students from more successful performance and progression. The paper then provides examples of successful or pilot mathematics reforms being trialled in other countries to shed light on and learn from practice overseas that is relevant to the Vorderman recommendations.

Methodology

"Prior to beginning research for this paper it was agreed with OCR that Scotland and Hong Kong would provide comparison case studies of overseas approaches to mathematics curriculum and assessment in secondary and upper secondary education"

Three different stages of research have taken place to inform this paper: desk research drawing together recent academic and policy literature on UK mathematics curriculum and assessment in secondary and upper secondary education; search and analysis of curriculum and research documents on secondary and upper secondary education in Scotland and Hong Kong; and a small number of interviews with key players in UK mathematics policy.

Background research

The desk research analysed existing academic and policy literature on mathematics curriculum and assessment in England. The research began by drawing together data and analysis on current curriculum and assessment mechanisms. Literature was then sought on approaches to reform of the mathematics curriculum and assessment including the Vorderman Report (2011) and expert policy papers from the Advisory Committee on Mathematics Education (ACME) and the Nuffield Foundation. The desk research helped draw out the policy context and the main areas of comparison between England, Scotland and Hong Kong including performativity, post-16 pathways and the relative merits of functional and academic curriculum and assessment content.

Case study research on approaches to the mathematics curriculum and assessment in Scotland and Hong Kong

In addition to the desk research on curriculum and assessment in England, targeted research was undertaken to obtain and analyse curriculum documents from Scotland and Hong Kong. Research documents were gathered from relevant government departments, research institutes and through the RSA

(Royal Society for the Encouragement of Arts, Manufactures and Commerce) and Oxford, Cambridge and RSA Examinations (OCR) contacts working in these countries. The curriculum documents gathered as a result of this research provided the material necessary to compare approaches in England with those of Hong Kong and Scotland, highlighting both areas of consensus and areas of difference.

Expert interviews

In order to provide further context and an overview of current debates on curriculum and assessment approaches in secondary and upper secondary mathematics education, four semi-structured interviews were undertaken with policy makers and experts working in this field in England and Scotland. Interviewees were asked about:

- Their perspective on the main areas of debate in relation to the mathematics curriculum and assessment
- Their perspective on the most effective reforms taking place either in England or elsewhere

Case study locations

Prior to beginning research for this paper it was agreed with OCR that Scotland and Hong Kong would provide comparison case studies of overseas approaches to mathematics curriculum and assessment in secondary and upper secondary education. These countries were chosen for a number of reasons. Both have recently undergone significant curriculum and assessment reform in order to address issues of attainment and progression. Both perform better than England in the mathematics scoring of the Programme for International Student Assessment (PISA) and Hong Kong is one the highest overseas performers in mathematics, ranking third in the most recent PISA scores (PISA, 2010) and identified as high-attaining by the Nuffield Foundation (Nuffield, 2010).

Scotland and Hong Kong also outperform England in relation to mathematics progression in upper secondary education. Both Scotland and Hong Kong fall into the middle bracket of countries in which between 21 – 50% of students continue studying mathematics in upper secondary education (in Hong Kong this is now 100% since mathematics has become compulsory as a result of educational reforms) compared to England which falls into the bottom bracket, in which only 15% of students continue to study mathematics (Nuffield, 2010).

On this basis both Scotland and Hong Kong were of interest as case study sites because of the educational reforms they have both undergone and their attainment and progression records. Although Scotland is not as high-attaining as Hong Kong, as a close neighbour that outperforms England on both attainment and progression, Scotland also provided an interesting comparison with our own approach to maths.

Context

"Many students do not realise that mathematical facility will be necessary for success in a range of higher education subjects"

(ACME, 2010)

This section first provides a brief overview of why mathematics is considered such an important subject by educationalists, policymakers and employers before summarising the most significant challenges affecting attainment and progression at secondary and upper secondary levels.

Why does maths matter?

1. Maths is needed to equip people with the skills they require for everyday tasks and to participate in the changing economy.

Basic numeracy and quantitative skills are increasingly necessary in all jobs for tasks including budgeting and data-handling. And the changing nature of the international economy means that maths skills and knowledge are in higher demand than ever. Science, Technology, Engineering and Mathematics (STEM) industries are becoming increasingly central to economic competitiveness and growth and will provide many of the jobs of tomorrow for young people (Royal Society, 2011). Enhancing mathematics qualifications and progression among English students must be part of any long-term strategy for UK economic growth and to ensure our young people are able to compete for jobs in an international market. Many students are currently unaware of the career advantages that strong quantitative skills will provide and 40% of employers have found that employees and prospective employees lack even basic numeracy skills that are necessary for work (CBI, 2009). Adequate financial literacy is also necessary for everyday tasks and decisions (e.g. budgeting or calculating loan repayments) that help people make responsible and informed financial decisions.

2. GCSE and post-16 qualifications are gatekeepers to a range of higher

education subjects including engineering, psychology, sciences and social sciences.

Many students do not realise that mathematical facility will be necessary for success in a range of higher education subjects (ACME, 2010). Subjects that require mathematical knowledge extend beyond mathematics and physics to include psychology, social sciences and human sciences (MacInnes, 2009). But the level of mathematical knowledge required in higher education courses is not always apparent from entry criteria or module descriptions, often because institutions worry they will not fill their courses if they include higher mathematics qualifications as an entry requirement (MacInnes, 2009). This leaves students who have given up mathematics in secondary education struggling to cope with the content of their degrees. Those universities that do explicitly require higher mathematics qualifications often use them as a filter by which to choose successful applicants since such qualifications are comparatively uncommon in England (ACME, 2011), again disadvantaging those students who have not studied mathematics since GCSE. Unless young people appreciate the continuing role that mathematics has – in obtaining entry to many higher education courses and in mastering course content – their aspirations to study degrees in subjects such as psychology or social sciences are compromised.

3. English universities are side-lining quantitative and mathematical content because students and staff lack the requisite confidence and ability. This has the potential to damage standards in English universities.

Improving the mathematics curriculum and assessment could also drive up standards in English universities. Some universities do not advertise the level of maths needed to comfortably study particular subjects for fear of hindering applications. Furthermore, recent research suggests that universities are marginalising mathematical content in the delivery of degree courses because English students are not capable of studying it or sometimes because the limited mathematical facility of teachers renders it difficult for them to teach advanced mathematical content (MacInnes, 2009). For instance in the social sciences, quantitative research methods may be neglected. It also means English universities are not keeping pace with international standards. It is common amongst universities overseas to require advanced mathematics qualifications prior to being accepted onto relevant degree programmes. In an increasingly international market, the failure to develop quantitative skills and content adequately has the potential to damage the standing of some English degrees amongst international students and to disadvantage English graduates in the global marketplace (Porkess, 2005).

40%

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have found that employees and prospective employees lack even basic numeracy skills

The challenges facing mathematics

A range of research studies have been undertaken to examine the problems that beset the study of mathematics in England. Drawing on some of the most recent studies, they conclude that broadly four factors influence attainment and progression. Firstly, a growing culture of performativity in schools; secondly, curriculum content that does not correspond to learner needs; thirdly, progression pathways that do not respond to learner needs; and finally a lack of specialist teachers (for example see: Vorderman, 2011; ACME, 2011; Royal Society, 2011).

Performativity and assessment

Assessment is an important element of measuring student success when combined with a focus on the process and value of learning itself. However a culture of performativity in England's schools has become routine with the annual publication of league tables that can detract from focusing on what is best for students. Performativity means that schools will prioritise credentials to maintain or improve league table positions. The decisions made to maximise school performance in league tables are not always in the interests of students and have in fact led to counter-productive practices (Williams, 2007). One such practice includes entering students who are on the borderline between a C and D GCSE grade (i.e. the borderline between passing and failing) into examinations multiple times and with multiple providers. Whilst this maximises the number of chances borderline students have to pass, this process does not necessarily help them learn and genuinely improve and can in fact give rise to or reinforce negative feelings about the subject.

Early entry is also common practice; in 2010 11% of students sitting GCSE Mathematics were 15 or younger (ACME, 2011). Students likely to gain a pass are often entered early in order for schools to 'bank' their grades and then direct learning efforts towards other subjects. Students who have 'banked' grades on the basis of early entry complete their study of mathematics at 14/15 years old and are potentially deprived of an even higher grade had they been given the opportunity to continue learning and sit their examination at 15 or 16 (ACME, 2011). Professor Alison Wolf discusses the prioritisation of institutional credentials over learner needs in detail in the recent Wolf Report (2011) and highlights the negative impact this is having on the mathematical knowledge and skills of English students.

Curriculum content

Course and examination content has been criticised for failing both struggling and high-attaining students alike. Students who struggle with mathematics in secondary schools need content that will support basic numeracy and be relevant for the workplace and everyday life. More able students need to be adequately prepared for the study of mathematics at A-level and beyond. Experts argue that GCSE Mathematics as currently conceived does not measure a 'mathematical way of thinking' and rewards accuracy over flair (Vorderman, 2011). Areas of maths which are more complex such as algebra, modelling, investigation and reasoning are not given sufficient teaching time and are even avoided to make time for more routine tasks necessary to pass tests. In the long-term this reduces the mathematical ability and confidence of students who need both time and challenges to grasp the subject and makes any transition to A-level all the more difficult. Content also fails to provide students with more basic numeracy and financial literacy skills according to the concerns of employer organisations including the CBI (CBI, 2009).

Pathways

In contrast to other OECD countries, only 15% of English students continue studying mathematics at A-level (Nuffield, 2010). The reasons for this centre around the limited number of post-16 pathways and the failure to design pathways that cater to a range of needs. Students who do not achieve a C in GCSE Mathematics during secondary education are often required to re-take their GCSE Mathematics (sometimes repeatedly) as a requirement for entry to upper secondary education at all. The emphasis on achieving a pass in GCSE Mathematics is understandable given the weight it carries in further education and the workplace. But for young people who have already struggled with GCSE Mathematics in secondary school, the prospect of repeating the same examination multiple times in upper secondary education risks further damaging their attitude to the subject.

Young people who do not achieve a B at GCSE can find it difficult to gain a place on a GCE mathematics course as a B grade is usually the minimum entry requirement for studying mathematics at A-level. Alternative qualifications for students who want or need to study mathematics post-16 but are unlikely or unable to study GCE Mathematics are scarce however, as Alison Wolf and others have noted (Wolf, 2011). The few existing non-GCE qualifications such as Free Standing Mathematics Qualifications (FSMQs) lack currency amongst employers and HE/FE institutions that prefer more traditional GCSEs and A-levels. There are no mathematics qualifications which specifically cater for vocational courses (ACME, 2011).

Teaching

The impact of high quality teachers is significant. A growing body of evidence shows that variation in teaching quality has a major impact on outcomes and that, all other things being equal, the difference between having an 'excellent' and a 'bad' teacher is equivalent to one GCSE grade (ippr, 2010). It is therefore concerning that there is such a shortage of secondary mathematics teachers with specialist knowledge (e.g. a maths degree). It is estimated that up to one in six secondary mathematics teachers have transferred from another subject and 25% of maths teachers have no post A-level qualification in related subjects (Vorderman, 2011). Alongside specialist knowledge, maths teachers must increasingly understand the needs of employers and teach with them in mind.

The way forward

The challenges for improving mathematics achievement and progression in England therefore centre on assessment, curriculum, pathways and teaching at two key periods of education: GCSE and upper secondary education. The following sections of this paper will examine routes for addressing these challenges. They will also compare the UK system with systems in Scotland and Hong Kong. England defies international norms on achievement and progression in mathematics, being one of only six OECD countries that fail to educate students in mathematics until they are 18 years old. As much of the drive for change has centred on the arguments that English students and institutions are being outperformed by international neighbours and the needs of the new international economy, there is value in examining how more successful countries approach mathematics education for 14 – 19 year olds. The Vorderman Report recommendation to make some form of mathematics compulsory until 18 should also be compared with overseas requirements.

Mathematics in secondary education

"The Smith (2004) and Tomlinson (2004) reports paved the way for rethinking the 14–19 mathematics curriculum with the aim of improving attainment and progression"

Before examining international approaches to reform of mathematics education, this section of the paper will chart the range of qualification and curriculum reforms in secondary and upper secondary mathematics that have already been trialled in England or are currently being proposed.

Secondary mathematics

The Smith (2004) and Tomlinson (2004) reports paved the way for rethinking the 14-19 mathematics curriculum with the aim of improving attainment and progression. Smith recommended that the then three-tier GCSE Mathematics was replaced by a two-tier system that provided all students with the opportunity to achieve a pass. Smith also encouraged discussions about making GCSE Mathematics a double award (similar to GCSE Science). A year later the Qualifications and Curriculum Authority (QCA) launched its *Mathematics Pathways Project*. This programme of work has mapped out new pathways for studying maths at both GCSE and A-level including improved offers for numeracy and functional mathematics. Recommendations include:

- When the pilot is complete, 'Use of Mathematics' qualifications (which have a focus on functional skills) should be rolled-out in order to create new learner pathways that will widen and increase participation and engagement in mathematics
- The development of more effective CPD for curriculum leaders in order to support the design of curricula, pathways and pedagogy which uses the available suite of mathematics qualifications
- Supporting high-achieving students to develop greater facility for algebra by age 16 and for this to be reflected and incentivised through assessment

- Further scrutiny of qualifications both in terms of analysis of assessment design and student responses (Noyes, 2010)

The new GCSE Mathematics 2010 corresponds to some of these recommendations including increased emphasis on functional elements of mathematics and problem solving. However there remains a consensus amongst experts and researchers that a single award is not fit for purpose and should be replaced by a double award as soon as possible (for example see: ACME, 2011; Vorderman, 2011; Noyes, 2010). Different models have been suggested for what a two-GCSE system might look like. The most common are outlined below.

Option 1: a simple double award

This option, advocated by the Smith Report (2004), would see GCSE Mathematics become a double award in the same vein as the old GCSE Science (Double Award) in which students are awarded two identical GCSEs (e.g. AA, BB, CC) to acknowledge the volume of work that had been undertaken to achieve the qualification. It makes little provision for curriculum content change or flexibility. This option was rejected by the QCA and other members of the mathematics community for a variety of reasons including a reduction of curriculum flexibility, the potential that students would assume that a far greater workload was involved in achieving a double award and the failure of a double award to recognise relative strengths (e.g. a student who is talented at functional maths but less capable at theoretical maths). QCA noted that for many of these reasons, the GCSE Science (Double Award) had proved unpopular and was being withdrawn (Noyes, 2010). This model has little support in the mathematics community today.

Option 2: GCSE 1 and GCSE 2

Proposed by Government in 2007, this option involved two separate maths GCSEs. GCSE 1 would be a gatekeeper qualification that incorporated functional numeracy skills used in everyday life but continued to support progression to study maths at A-level and beyond. GCSE 2 would be a further qualification aimed at high-achieving and motivated students. GCSE 2 was partly developed in response to the criticism that the transition between GCSE and A-level mathematics is currently too vast and that the algebra facility of high-achieving students is not adequate (Noyes, 2010). Although piloted in schools, the model was never adopted and received criticism from experts and media alike (for example see: BBC, 2007). The Advisory Committee on Mathematics Education (ACME) and the QCA both questioned whether GCSE 1 would be accepted by colleges as a gatekeeper qualification for A-level mathematics because the academic focus of GCSE 2 would make it the more natural stepping-stone to advanced study. There was also concern that some schools would only offer GCSE 1 – the 'easier' option – in order to maximise league table performance.

Option 3: a linked pair/ twin GCSEs

Since September 2010, twin GCSEs in *Applications of Mathematics* and *Methods in Mathematics* have been piloted by four examinations boards.[1] Proficiency in basic techniques is required in both courses so they share roughly 30% common content. The remaining syllabus for *Applications* is designed to address the lack of functional maths skills and engagement among students by addressing real world problems and supporting the development of skills such as data handling and financial literacy. Examples include being able to carry out enterprise calculations including appreciation and depreciation; using mathematics in the context of domestic finance including loan repayments and exchange rates; and using probability to estimate risk (OCR, 2010). The *Methods* syllabus focuses on more formal content including algebra, reasoning and geometry and provides the foundations students will usually need to study mathematics at A-level. Examples of curriculum content include using algebra to construct and support arguments; using vectors; and understanding and using circle theorems (OCR, 2010).

Both qualifications are designed to be of equal difficulty, both offer foundation and higher papers and both are assessed solely by examination. But the qualifications emphasise and develop different skills sets. The expectation is that the majority of students will take both qualifications. The linked pair has been a popular option amongst the mathematics community and received the endorsement of the Vorderman Report (2011).

Only
15%
of students
continue studying mathematics in upper secondary education

Upper secondary mathematics

Only 15% of students continue studying mathematics in upper secondary education, in stark contrast to the majority of OECD countries (Nuffield, 2010). This is in no small part because pathways for mathematics after secondary school are so limited. Students who fail to gain a pass in secondary education are often consigned to re-taking their GCSE in order to gain entry to upper secondary education. The main route for continuing the study of mathematics is GCE Mathematics and GCE Further Mathematics, both of which are studied by high-achieving students who have a strong interest in the subject and might well choose to study mathematics in higher education. The options are limited for students re-taking their GCSE or for students who have no wish to study an academic maths A-level as a precursor to higher education and do not cater for a range of ability levels or career aspirations (ACME, 2011). Debate in England is therefore centred on how pathways for different levels of ability and interest in mathematics might be better catered for in upper secondary education.

GCE Mathematics and GCE Further Mathematics

The existing GCE Mathematics and GCE Further Mathematics focus on academic mathematics as a precursor to studying mathematics or related subjects in higher education. GCE Mathematics includes compulsory units in pure maths with options to study statistics or mechanics. GCE Further Mathematics introduces a range of new and more complex concepts including hyperbolic functions, complex numbers, polar coordinates, and differential equations. Overall these qualifications are viewed as appropriate for the students they are serving although ACME has recommended that they place greater emphasis on problem-solving, modelling and the construction and communication of rigorous mathematical arguments to ensure students are well-prepared for mathematics in higher education (ACME, 2011).

GCE Use of Mathematics

Currently being piloted, GCE Use of Mathematics has been designed to meet the needs of students who want to continue studying mathematics in upper secondary education but have not achieved a high enough grade to study GCE Mathematics and/or GCE Further Mathematics. In the past, students with a C at GCSE and students who have found the traditional AS course too demanding have stopped studying mathematics completely. GCE Use of Mathematics provides a new pathway for these students, providing them with an opportunity to study mathematical topics in practical, real-life contexts. Curriculum content includes key concepts, personal finance and decision mathematics (AQA, 2008) and has reduced academic mathematics content. Assessment has a greater emphasis on coursework than GCE Mathematics.

There is some evidence that positive feedback received from other students is persuading potential AS drop-outs to transfer to GCE Use of Mathematics (EMP, 2011). This qualification also provides a clearer pathway for students who plan to study related subjects in further or higher education that will require some mathematics but do not require GCE Mathematics, e.g. psychology. However, the relationship between assessment and pedagogy has not settled and has produced some unintended consequences, for instance one pilot centre has reported the perceived lack of basic skills in the students means that teaching has become more traditional over the pilot period, with basic skills being taught separately and not through working in context (EMP, 2011). There is also a need for better communication with universities about the value of GCE Use of Mathematics; otherwise there is a danger that the qualification will lack the currency and kudos of the more traditional GCE Mathematics in subjects that will require some mathematics but do not require GCE Mathematics.

Free-standing Mathematics Qualifications (FSMQs)

Free-standing Mathematics Qualifications are a suite of mathematics qualifications available at level 1, 2 and 3. They were initially designed as a means of meeting the needs for mathematics units as part of pre-vocational programmes but were quickly identified as a qualification that would be equally useful to support students on academic programmes (Noyes, 2010). In this way they have been used to bridge the gap between GCSE and A-level mathematics and can typically be taken by high-achieving students who have completed their GCSE early. This provides a pathway for high-attaining students to continue mathematics education and ease their transition into upper secondary mathematics. FMSQs at levels 1 and 2 can be used in combination to obtain a GCSE in Use of Mathematics. This was a useful pathway for students who had failed to gain a pass in GCSE Mathematics at key stage 4 and for students who would not wish to progress to level 3 study of mathematics and would find the focus on applications valuable.

However, Ofqual have recently decided that after the completion of the pilot period this qualification can no longer be designated a GCSE and will instead be offered as a Certificate in the Use of Mathematics. Experts including ACME have argued that this is a serious blow to post-16 pathways. This is because the Certificate qualification will likely struggle in the marketplace without the kudos of the GCSE label and its role as a gatekeeper to subsequent opportunities in HE and the workplace (Noyes 2010).

The Vorderman Report (2011) recommended the implementation of a Mature GCSE available for post-16 students who have not achieved a C grade or above in mathematics as an alternative to repeating the same GCSE Mathematics curriculum studied in secondary education. The intended focus of a Mature GCSE on functional and other more vocational elements of mathematics makes this recommendation similar to the GCSE previously offered through FSMQs level 1 and 2.

Maths for citizenship

The Vorderman Report (2011) also recommends the creation of a maths pathway for students who are taking qualifications that involve no mathematics, e.g. three or four humanities A-levels, in recognition of the important role mathematics has for all upper secondary students in developing and maintaining numeracy and financial literacy skills. This course would be designed to ensure all students continue the study of mathematics in forms that will be useful to the workplace (so there would likely be a strong focus on functional skills) and everyday tasks (e.g. financial maths, data handling). There seems to be some commonality between this and the advanced FSMQ qualification.

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1. The four awarding organisations piloting the GCSE Mathematics linked pair are AQA, Edexcel, OCR and WJEC.

International case studies

This section of the report will offer international perspectives from Scotland and Hong Kong on reform of mathematics education in secondary and upper secondary education with a particular emphasis on reforms made to widen student pathways and broaden the application of mathematics. The case studies will start by outlining the state of mathematics curriculum and assessment in secondary education before moving on to chart the landscape in upper secondary education in both Scotland and Hong Kong.

Case study: secondary education curriculum and assessment in Scotland

Scotland's education system has recently undergone a radical overhaul to provide teachers and students with more freedom and flexibility. New guidance for classroom teaching known as the 'Curriculum for Excellence' was introduced into schools in September 2010. The Curriculum for Excellence is less prescriptive than the previous curriculum and has a new emphasis on inter-disciplinary learning and personalisation (Scottish Government, 2008). New exams are due to be introduced in 2012/13 and in 2013/14 and existing exams will be revised to fit in with the new curriculum. The redevelopment of the curriculum and assessment system in Scotland was motivated by similar challenges to those faced in England, including the need to make learning more engaging and less performative; the need to provide clearer progression pathways through secondary and upper secondary education; the arguable need to provide a better balance between academic and vocational knowledge and skills; and to ensure assessment and certification better meet the variety of learner needs (Scottish Government, 2008). At S4 and S5, the nearest equivalent to England's years 10 and 11 (i.e. the years spent studying towards GCSE qualifications), the old system of Standard Grade and Intermediate 1 and 2 is being replaced with National 4 and National 5 in order to promote greater flexibility and attainment (Black, 2009).

National 4 and National 5

As in England, the Scottish qualification system for students finishing secondary education has been criticised for a lack of flexibility. National 4 and National 5 are the Curriculum for Excellence's answer to these criticisms. Replacing the existing Standard and Intermediate qualifications in 2013, National 4 and National 5 aim to provide an assessment framework which supports curriculum learning rather than leading it. Students will begin studying towards National 4 and National 5 at 14/15 and will usually study up to eight subjects including mathematics. The approach is known as 'bi-level', allowing students to be taught at different speeds and progress when they are ready. For some students, this might involve working towards National 4 for two years, whilst the most able students will be able to bypass National 4 and National 5 altogether and begin studying for their Higher in their best subjects (Scottish Government, 2011). This system allows students to delay decisions about which level to sit examinations at for longer (students who perform better than expected might sit National 5 rather than National 4), which encourages students to aim for the highest possible grade and prevents students from being pigeon-holed too early.

The Mathematics Higher focuses on academic maths including **more complex** elements of the National 5 curriculum

Perhaps most radically, National 4 is not assessed via an external exam but by teachers through a combination of observation and coursework. This means students do not have to plan their learning around assessments; one of the challenges experienced in the high-stakes testing that dominates the English system. This could provide space for young people to have a more focused and personalised learning experience which could also increase learner motivation (SQA, 2011). This is particularly significant for mathematics, where too often England's current approach to mathematics study in secondary education damages motivation and engagement amongst learners. The user-centric approach also means there is a more staged route to mathematics for higher-achieving students, making the step from National 5 to Higher Mathematics easier than the transition between GCSE Mathematics and A-level mathematics.

The structure and curriculum of mathematics qualifications at National 4 and National 5 is arranged into two distinct qualifications: National 4/5 Mathematics and National 4/5 Lifeskills Mathematics. Both qualifications are of equal value but emphasise different uses of mathematics. This division is similar to the linked pair GCSEs being piloted in England in that it distinguishes between functional mathematics and academic mathematics. National 4/5 Mathematics includes mainly academic maths subjects including the use of algebra (e.g. evaluating an expression or formulae that has more than one variable) and geometry (e.g. using rotational symmetry) whilst National 4/5 Lifeskills Mathematics has a greater focus on personal maths (e.g. converting between currencies) and workplace-relevant maths (e.g. comparing data sets) (SQA, 2011). National 4/5 Lifeskills Mathematics is intended as an 'exit' qualification which provides students with sufficient mathematical proficiency for the workplace and everyday situations (SQA, 2011). Because both National Qualifications are designed to be of equal difficulty and both result in the award of a National 4 or National 5 they might avoid the issues of kudos and hierarchy that have plagued prior attempts to split mathematics qualification in two in England.

Case study: secondary education curriculum and assessment in Hong Kong

Recent reforms to the Hong Kong education system have eradicated the distinction between secondary and upper secondary education. The New Senior Secondary (NSS) academic structure involves all students studying towards one set of examinations at 17 years old, then starting work or entering four year universities courses (HKEAA, 2010). Students begin studying for the NSS at 14/15. Prior to this, students study three years of junior secondary education which equates to England's key stage 3 (years 7, 8 and 9). The NSS is provided as a case study in the section of this paper which addresses upper secondary education and will not be addressed here.

The junior secondary curriculum and assessment framework is not directly comparable to GCSEs or National 4/5 as it is directed at a younger age group. However it is worth noting that even in junior secondary education the aims and objectives of the curriculum include that mathematics taught and learned should be applicable and relevant to future studies, workplaces and everyday lives, again emphasising the need for strong functional and academic content. There is also an emphasis on catering to different needs and interests through the creation of foundation and non-foundation content. Foundation content is basic mathematical knowledge and concepts that all students need and non-foundation content can be chosen with the needs of individual students or groups of students in mind. A certain degree of flexibility combined with a core of minimum content mirrors Scottish approaches to secondary mathematics education.

Case study: upper secondary education curriculum and assessment in Scotland

Post-16 pathways in Scotland are more homogenous than England's offer and will change little under the Curriculum for Excellence because of their popularity and relative success. Like England, mathematics in Scotland is not compulsory at the completion of formal secondary education. However, roughly 26% of Scottish students continue studying mathematics as part of the Scottish Higher, considerably more than the 15% of students who study mathematics post-16 in England (Nuffield, 2010).

Higher and Advanced Higher

The Mathematics Higher focuses on academic maths including more complex elements of the National 5 curriculum such as algebra, calculus, working with statistics and the use of probability. Although the Higher is not undergoing significant change as a result of the Curriculum for Excellence reforms, each Unit will include an explicit focus on the applications of learning to real-life including the workplace.

The higher progression rates for mathematics in Scotland are not due to more diversity in the qualifications available (there is no 'Mathematics for Citizenship' style certificate for instance). All students study mainly academic mathematics but where possible curriculum content relays how academic mathematics might be used in real-life. Some research has suggested the more homogenous provision actually incentivises up-take because the system is easier for students to understand and the value of the Higher qualification as a gatekeeper to higher education well understood (Nuffield, 2010). However it is likely that the higher progression is also due to the wider range of subjects taken at Higher in Scotland. Students typically study towards five Highers which gives more opportunity for breadth (Nuffield, 2010). The more flexible approach to qualifications leading up to the Higher – both the current Intermediate qualifications and the proposed National 5 – also provide a smoother transition to the study of more advanced mathematics than is currently available in England.

Scottish Certificate for Numeracy

Curriculum for Excellence also brings a sustained focus on developing literacy and numeracy skills amongst Scotland's young people. To help strengthen this focus, the Scottish Government is proposing new separate awards to accredit young people's literacy and numeracy skills – the Scottish Certificate for Literacy and the Scottish Certificate for Numeracy. The awards will be available at SCQF levels 3 to 5 to ensure students of different abilities can gain qualifications and have their efforts recognised. The expectation is that all young people will be presented for these awards unless there are exceptional reasons for not doing so (Scottish Government, 2011). The intention is also to ensure that the structure of these awards is flexible enough to make them available to adult learners. The focus of the Numeracy certificate is basic and life-skills mathematics. Skills that will be assessed include:

- basic number processes – four operations (addition, subtraction, division and multiplication), percentage, simple fractions;
- understanding time, timetables, time management and social implications of punctuality;
- money – using, managing, planning, earning;
- information handling – sourcing (electronically and physically), interrogating, processing and decision making; and
- chance and uncertainty – concept of probability, how statistics can be misleading, probability of events happening e.g. towns being flooded due to climate change (Scottish Government, 2011)

The Numeracy certificate will be flexible in terms of assessment, drawing on evidence from a student's work across the curriculum and an externally assessed examination. For adult learners, the element of internal assessment would draw upon work completed in college, employment or other activities. This qualification will provide support for students or adult learners who have struggled in education or want to provide certification of numeracy for employment (Tierney, 2009). However, this qualification might suffer from similar kudos difficulties to England's Free-standing Mathematics Qualifications (FSMQs) which are unlikely to be highly-regarded by employers and higher educational institutions compared to mainstream mathematics qualifications (in this case, National 4/5).

The New Senior Secondary structure

Under the new academic system, all Hong Kong secondary students are expected to complete three years of junior secondary education followed by three years of senior secondary education. Students are required to study four core subjects (Chinese language, English language, mathematics and liberal studies), and one to three elective subjects from a total of 20 subject areas. Students can also study 'other learning experiences' including aesthetic, physical and careers education, and community service. Students can also take pre-vocational 'applied learning courses' that correspond to local labour market needs as elective subjects in Secondary 5 and Secondary 6 (final two years).

As a percentage of total classroom instruction, core subjects will consume 45-55% of the curriculum, elective subjects 20-30%, and Other Learning Experiences (moral and civic education, community service, physical development, aesthetic development, and work experience) 15-35% depending on the school and student.
(CDC, 2007)

Case study: upper secondary education curriculum and assessment in Hong Kong

In 2009 Hong Kong overhauled the academic structure of senior secondary education, introducing the New Senior Secondary (NSS) academic structure and curriculum. Under this system, secondary school students take only one round of final examinations (rather than examinations at 15/16 and 17/18) and graduate from upper secondary education a year earlier at age 17 (HKEAA, 2010). As a result Hong Kong universities now provide four-year undergraduate degrees in lieu of British-modelled three-year degrees. The new 3+3 academic structure replaced a 5+2 structure similar to England's. The decision to hold public examinations only once in this period, rather than twice as in England and Scotland allows students to focus on learning and knowledge development and to avoid the performativity that otherwise dominates learning (Chan, 2010). Indeed, new curriculum guidelines for the NSS specifically state that 'assessment should be recognised not only as a means to gauge performance but also to improve learning' (HKEAA, 2010).

The NSS makes mathematics compulsory for all students because of the subject's economic importance and the life-skills and broader problem-solving abilities it helps students develop. Despite the homogeneity of this qualification, the NSS nonetheless strives to cater to diverse learner needs and pathways through a flexible curriculum and approach to assessment. This involves all students studying compulsory subjects which provide academic and functional mathematical knowledge that is applicable both in further study and the workplace. However, there is also an optional extended part to mathematics study in the NSS which provides an explicit pathway for students who plan to study mathematics or related subjects in higher education.

Compulsory Part

The Compulsory Part serves as a foundation for all students and provides flexibility to cater for the diverse needs and abilities of individual students. Three subject areas are 'numbers and algebra', 'measures, shape and space' and 'data handling' and typical curriculum content includes (HKEAA, 2008):

- manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve real-life problems and justify the validity of the results obtained (Numbers and algebra)
- formulate and write geometric proofs involving 2-dimensional shapes with appropriate symbols, terminology and reasons (Measures, shape and space)
- integrate the knowledge in statistics and probability to solve real-life problems (Data handling) (CDC, 2007)

The Compulsory Part purposely blurs the academic and practical demarcation to ensure all students have a diverse range of associated skills. Each subject area includes academic and applications content that aims to provide students with understanding of essential mathematics concepts alongside the ability to put this knowledge to use in the world, e.g. to use reasoning to solve problems or approach financial decisions (CDC, 2007). The emphasis on the value of academic as well as functional skills for *all* students regardless of ability is in contrast to England and Scotland where the emphasis has been on the importance of qualifications that prioritise functional skills for less able students.

Extended Part

Students who are more able in mathematics or need more mathematical knowledge and skills to prepare for their future studies and careers can choose to extend their study to include a module from the Extended Part. Module 1 (Calculus and Statistics) focuses more on mathematical applications, whereas Module 2 (Algebra and Calculus) places more emphasis on mathematical concepts and knowledge (CDC, 2007). Students who would like to learn more mathematics may choose the module which best suits their interests and needs.

The NSS requires all students to study mathematics until higher education, further education or employment commences. It provides only one qualification for this that must be studied by all students: the Hong Kong Diploma of Secondary Education. This avoids problems of kudos that have plagued non-GCSE or A-level qualifications such as FSMQs and Certificates in England.

However, the Hong Kong system is mindful of addressing diverse learner needs and thus allows reasonable flexibility within the Diploma curriculum to support different learner pathways.

> "The NSS makes mathematics compulsory for all students because of the subject's economic importance and the life-skills and broader problem-solving abilities it helps students develop"

Discussion

> "The Scottish model echoes the linked mathematics GCSEs currently being piloted in England which has achieved strong support amongst experts and positive early feedback from participating schools"
>
> (Noyes, 2010)

This section of the report brings together debates about mathematics reform in England – reinvigorated by the Vorderman Report (2011) – and evidence from approaches to mathematics education in Scotland and Hong Kong before offering a series of recommendations for moving forward.

Functional and academic skills

Ensuring that qualifications are built around learners' needs is crucial to learner engagement and attainment in secondary mathematics education. There is relative consensus that supporting the development of both functional and academic skills should be part of any GCSE Mathematics offer. Junior secondary education in Hong Kong incorporates both functional and academic content for all students. The favoured approach in Scotland is splitting mathematics into two separate qualifications, with one focusing on academic mathematics and the other on functional or 'life' skills. The expectation here is that the brightest mathematics students would be encouraged to study for both qualifications and that Lifeskills Mathematics provides other students with adequate mathematical skills for employment and everyday situations. This is likely to be of particular appeal to employers who have previously criticised the limited mathematical knowledge of employees (CBI, 2007). The Scottish model echoes the linked mathe matics GCSEs currently being piloted in England which has achieved strong support amongst experts and positive early feedback from participating schools (Noyes, 2010).

Flexible and bi-level qualifications

User-centred qualifications that are focused on learning rather than performativity can also be delivered through a more flexible approach to assessment. This could involve a coursework and in-class assessment approach (as per the Scottish National 4 qualification) for learners who need more time and support with mathematics. Bi-level pathways which allow learners to move between qualifications depending on the extent of their progress (as with the flexible approach to moving between National 4 and National 5) allow room for improvement amongst students and ensure that all students are able to achieve the highest grade possible. Increased flexibility also gives more space for students to develop and demonstrate mathematical 'flair' – something experts have suggested is on the decline amongst English secondary school students (Vorderman, 2011).

Compulsory mathematics until 18

High numbers of students studying mathematics in upper secondary education is positive for the development of life and employment skills and for wider economic goals as STEM industries become increasingly important. Making mathematics compulsory in upper secondary education (i.e. level 3 in England) is one means of ensuring all students gain some advanced mathematical knowledge (as per Hong Kong's Diploma of Secondary Education). The Vorderman Report (2011) recommends compulsory mathematics in upper secondary education in some form but falls short of suggesting that all students should study GCE Mathematics, instead recommending the creation of multiple qualifications including a Mature GCSE and a Maths for Citizenship qualification. Although the dynamics and details of what mathematics in upper secondary education should look like (discussed further below) are still open to debate there does appear to be an emerging consensus that it is valuable for all students to study mathematics in some form post-16.

Diversity vs. homogeneity

The cases of Hong Kong and Scotland demonstrate that diversity in qualifications is not necessarily the most effective route to increasing the study or knowledge of mathematics study in upper secondary education, however. In fact, the homogeneity of the Scottish Higher qualification has been cited as one of the reasons for the higher levels of progression, making it easier for students, employers and higher education institutions alike to understand the relative value of qualifications (Nuffield, 2010).

However, in both Hong Kong and Scotland, upper secondary mathematics qualifications do provide diversity in curriculum content. The Higher allows students to study the application of mathematics to real-life and the Hong Kong Diploma allows high-attaining students to specialise in calculus or algebra through the Extended Part. The key point being that in both cases – Hong Kong and Scotland – qualifications are available at different levels and incorporate relevance to the real-world in all qualifications, leaving mathematics in these countries more flexible and less niche.

Mathematical literacy and arithmetic

It is also noteworthy that in Scotland and Hong Kong all upper secondary mathematics qualifications emphasise literacy in basic academic mathematics subjects including arithmetic as well as functional mathematics for all students. This point should not be lost in the emphasis on workplace-relevant skills in the UK; basic arithmetic and other academic mathematics are arguably an important feature of course content for all upper secondary students and particularly for those who have not achieved GCSE Mathematics. Those overseeing the post-16 qualification landscape – including mooted new qualifications such as the Mature GCSE and a Maths for Citizenship qualification – should consider how they can incorporate and deliver these skills too.

Conclusion

Mathematics knowledge and qualifications are increasingly important gateways to further and higher education, for crucial life-skills and in order to respond to economic change. But the way mathematics is taught and assessed in England has not always kept pace with these changes or with the needs of learners and has left one in four adults functionally innumerate (Vorderman, 2011). This decline is by no means an international norm; mathematics attainment and progression in growing economies such as China and India increases year on year (Kuenzi, 2008) and even close neighbour and case study site Scotland manages to encourage ten per cent more of its students into upper secondary mathematics (Nuffield, 2010) than England.

The underlying problems are that mathematics curriculum and assessment have not always been designed around learner or broader economic and educational needs. There have been few upper secondary pathways for students that fail outright or fail to gain a B in GCSE Mathematics. Students who enter upper secondary education without a pass in GCSE Mathematics are often required to re-take their GCSE, inspiring a lifelong dislike for the subject. And there has not been a clear enough distinction between functional and academic mathematics. The challenge is to build systems of content and assessment in secondary and upper secondary education that cater for diverse needs, provide essential numeracy and arithmetic skills for work and citizenship and provide all students with knowledge and qualifications that can be used to access education and employment opportunities.

Much work has already been undertaken to redress some of these problems. As long ago as 2004 Adrian Smith identified the need for more emphasis on functional skills. The recent Vorderman Report has restated that need and provided new thinking on what pathways could look like. The diversification of qualifications to include FSMQs and pilots of linked Mathematics GCSEs and GCE Use of Mathematics highlight that some of this change is already underway.

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We have seen in this paper how countries with higher rates of progression and attainment are responding to challenges around mathematics. While England cannot and should not simply import models that appear to work overseas, we can identify the underlying principles and approaches that make for better mathematics knowledge and skills among populations. The case studies in this paper and preceding discussion in this section of the report highlight some of the approaches that might inform or give strength to our own reforms:

- The need to provide flexible, bi-level qualifications that allow students to progress at their own speed and aim for the highest possible grade (as per National 4 and 5 in Scotland). On this basis the linked pair of mathematics GCSEs currently being piloted should be rolled out across England.
- Upper secondary education should extend beyond re-takes even for students who have not gained GCSE Mathematics.
- Further consideration of making mathematics compulsory for all students in upper secondary education is needed and should draw on Hong Kong's NSS experience.
- The importance of incorporating functional *and* academic elements (including arithmetic) in secondary and upper secondary education.
- Designing assessment arrangements that limit performativity, for instance through some element of teacher-led assessment for students studying foundation-level qualifications.
- Recognising that too much diversity in qualification-type can be confusing for students, education-providers and employers and that diversity in curriculum content can be a better route for meeting diverse learner needs (as with the Scottish Higher and Hong Kong's NSS).
- Once the pilot and evaluation of the linked pair GCSEs is complete, the mathematics community should come together over a one or two day period to consider the big picture and learning from overseas, and develop definitive recommendations for the future of mathematics education in England.

The recent flurry of policy activity on mathematics, and in particular the Vorderman Report, shows that our 'problem with maths' continues to preoccupy policymakers and educationalists. The ideas and examples from home and overseas that are outlined in this paper should provide some food for thought, in some instances strengthening the case for intended reforms and in others providing new and sometimes challenging thinking. Both will be necessary to realise the goal of creating a 'world class mathematics education for all'.

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